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Determination of $|V_{cb}|$ from Inclusive Decays $B \to X_c \ell \nu$ using a Global Fit

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1 Introduction

In this article, we review the theory and the experimental status of inclusive semileptonic B meson decays $B \to X_c \ell \nu$. Based on these inputs, we present the latest determination of the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ [1] and of the b-quark mass m_b, obtained by the Heavy Flavor Averaging Group (HFAG).

2 Theory

The theoretical tool for calculating the inclusive semileptonic B decay width $\Gamma(B \to X_c \ell \nu)$ is the Operator Product Expansion (OPE) [2, 3],

$$\Gamma(B \to X_c \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left(1 + \frac{c_5(\mu)\,\langle O_5 \rangle(\mu)}{m_b^2} + \frac{c_6(\mu)\,\langle O_6 \rangle(\mu)}{m_b^3} + \mathcal{O}(\frac{1}{m_b^4})\right) \ . \quad (1)$$

At the leading order in $1/m_b$, this expression coincides with the parton model result, *i.e.*, the decay width of a free b-quark. At higher orders appear Wilson coefficients (c_5, c_6) multiplying expectation values of local operators $(\langle O_5 \rangle, \langle O_6 \rangle)$. The coefficients c_5 and c_6 contain the perturbative QCD physics while the local operators are non-perturbative objects.

Equation 1 assumes parton-hadron duality, which implies that the hadronic parameters $\langle O_5 \rangle$ and $\langle O_6 \rangle$ do not depend on the final state of the decay. Thus, they

also appear in OPEs for other inclusive B meson observables and can be measured in experiments. This entire procedure is referred to as a global fit.

The inclusive observables used in this analysis are the (truncated) moments of the lepton energy E_ℓ (in the B rest frame) and the m_X^2 spectra in $B \to X_c \ell \nu$, where m_X^2 is the invariant mass squared of the hadronic system X_c accompanying the lepton-neutrino pair. The lepton energy moments are defined as

$$\langle E_\ell^n \rangle_{E_{\rm cut}} = \frac{R_n(E_{\rm cut})}{R_0(E_{\rm cut})} \ , \quad R_n(E_{\rm cut}) = \int_{E_\ell > E_{\rm cut}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell \ , \tag{2}$$

where $E_{\rm cut}$ is the lower lepton energy threshold and $d\Gamma/dE_\ell$ is the partial semileptonic width as a function of the lepton energy. The hadronic mass moments are

$$\langle m_X^{2n} \rangle_{E_{\rm cut}} = \frac{S_n(E_{\rm cut})}{S_0(E_{\rm cut})} \ , \quad S_n(E_{\rm cut}) = \int_{E_\ell > E_{\rm cut}} m_X^{2n} \frac{d\Gamma}{dm_X^2} dm_X^2 \ . \tag{3}$$

Also here, the integration is over the $B \to X_c \ell \nu$ phase space restricted by the requirement $E_\ell > E_{\rm cut}$.

OPE calculations of the semileptonic width and these moments have been obtained in two theoretical frameworks, referred to by the name of the renormalization scheme used for the quark masses. The calculations in the *kinetic scheme* are now available at next-to-next-to-leading (NNLO) order in α_s [2, 4]. At leading order in the OPE, the non-perturbative parameters are the quark masses m_b and m_c. At $\mathcal{O}(1/m_b^2)$ ($\mathcal{O}(1/m_b^3)$) the parameters are μ_π^2 and μ_G^2 (ρ_D^3, ρ_{LS}^3). Independent expressions are available in the *1S scheme* [3]. Here, the long-distance parameters are m_b at leading order, λ_1 and λ_2 at $\mathcal{O}(1/m_b^2)$ and ρ_1, τ_{1-3} at $\mathcal{O}(1/m_b^3)$. Note that the numerical values of the quark masses in the two schemes cannot be compared directly due to their different definitions. The expressions used here include power corrections up to $\mathcal{O}(1/m_b^3)$ though higher orders have already been calculated [5].

3 Experiment

The most precise measurements of partial semileptonic branching fractions and moments in $B \to X_c \ell \nu$ are obtained by the Belle [6, 7] and BaBar collaborations [8] analyzing 152 and 232 million $\Upsilon(4S) \to B\overline{B}$ events, respectively. These studies proceed as follows: first, the decay of one B meson in the event is fully reconstructed in a hadronic mode ($B_{\rm tag}$). Then, the semileptonic decay of the second B meson ($B_{\rm sig}$) is identified by searching for a charged lepton among the remaining particles in the event.

The observed E_ℓ and m_X^2 spectra are distorted by resolution and acceptance effects. Belle corrects for this by unfolding the observed spectra using the Singular Value Decomposition (SVD) algorithm [9] and measures the energy moments $\langle E_\ell^k \rangle$

Table 1: Experimental inputs for the global analysis of $B \to X_c \ell \nu$. n is the order of the moment, c is the threshold value in GeV. In total, there are 29 measurements from BaBar, 25 measurements from Belle and 12 from other experiments.

Experiment	Hadron moments $\langle M_X^n \rangle$	Lepton moments $\langle E_\ell^n \rangle$	Photons moment $\langle E_\gamma^n \rangle$
BaBar	$n=2$, $c=0.9, 1.1, 1.3, 1.5$ $n=4$, $c=0.8, 1.0, 1.2, 1.4$ $n=6$, $c=0.9, 1.3$ [8]	$n=0$, $c=0.6, 1.2, 1.5$ $n=1$, $c=0.6, 0.8, 1.0, 1.2, 1.5$ $n=2$, $c=0.6, 1.0, 1.5$ $n=3$, $c=0.8, 1.2$ [8, 10]	$n=1$, $c=1.9, 2.0$ $n=2$, $c=1.9$ [12, 13]
Belle	$n=2$, $c=0.7, 1.1, 1.3, 1.5$ $n=4$, $c=0.7, 0.9, 1.3$ [7]	$n=0$, $c=0.6, 1.0, 1.4$ $n=1$, $c=0.6, 0.8, 1.0, 1.2, 1.4$ $n=2$, $c=0.6, 1.0, 1.4$ $n=3$, $c=0.8, 1.0, 1.2$ [6]	$n=1$, $c=1.8, 1.9$ $n=2$, $c=1.8, 2.0$ [14]
CDF	$n=2$, $c=0.7$ $n=4$, $c=0.7$ [15]		
CLEO	$n=2$, $c=1.0, 1.5$ $n=4$, $c=1.0, 1.5$ [17]		$n=1$, $c=2.0$ [16]
DELPHI	$n=2$, $c=0.0$ $n=4$, $c=0.0$ [18]	$n=1$, $c=0.0$ $n=2$, $c=0.0$ $n=3$, $c=0.0$ [18]	

for $k = 0, 1, 2, 3, 4$ and minimum lepton energies ranging from 0.4 to 2.0 GeV. Moments of the hadronic mass $\langle m_X^k \rangle$ are measured for $k = 2, 4$ and minimum lepton energies from 0.7 to 1.9 GeV. BaBar applies a set of linear corrections, which depend on the charged particle multiplicity of the X system, the normalized missing mass, $E_{\text{miss}} - p_{\text{miss}}$, and the lepton momentum. In this way, BaBar measures the moments of the hadronic mass spectrum up to $\langle m_X^6 \rangle$ for minimum lepton energies ranging from 0.8 to 1.9 GeV. In Ref. [8] the earlier measurement of the lepton energy moments in $B \to X_c \ell \nu$ [10] is updated using new branching fraction measurements for background decays and an improved evaluation of systematic uncertainties.

All measurements used by HFAG for determining $|V_{cb}|$ inclusive are listed in Table 1. The only external input for the fit is the average lifetime τ_B of neutral and charged B mesons, taken to be (1.582 ± 0.007) ps [11].

4 Results for $|V_{cb}|$ inclusive and m_b

By fitting the measurements in Table 1 to the OPE expressions of the semileptonic width and of the $B \to X_c \ell \nu$ moments, properly accounting for correlations in the theory expressions and experimental data, $|V_{cb}|$, the b-quark mass and the other hadronic parameters are obtained. The moments in $B \to X_c \ell \nu$ are sufficient for determining $|V_{cb}|$ but measure the b-quark mass only to about 50 MeV preci-

Table 2: Global fit results in the kinetic scheme for different constraints.

Constraint	$\|V_{cb}\|$ (10^{-3})	$m_b^{\rm kin}$ (GeV)	μ_π^2 (GeV^2)	χ^2/d.o.f.
$B \to X_s\gamma$	$41.94 \pm 0.43_{\rm fit} \pm 0.59_{\rm th}$	4.574 ± 0.032	0.459 ± 0.037	$27.0/(66-7)$
$m_c^{\overline{\rm MS}}$(3 GeV)	$41.88 \pm 0.44_{\rm fit} \pm 0.59_{\rm th}$	4.560 ± 0.023	0.453 ± 0.036	$33.4/(55-7)$

Table 3: Global fit results in the $1S$ scheme for different constraints.

Constraint	$\|V_{cb}\|$ (10^{-3})	m_b^{1S} (GeV)	λ_1 (GeV^2)	χ^2/d.o.f.
$B \to X_s\gamma$	41.96 ± 0.45	4.691 ± 0.037	-0.362 ± 0.067	$23.0/(66-7)$
None	42.37 ± 0.65	4.622 ± 0.085	-0.412 ± 0.084	$13.7/(55-7)$

sion. Therefore, additional constraints are introduced: the photon energy moments in $B \to X_s\gamma$, or a precise constraint on the c-quark mass. For the former, calculations of the $B \to X_s\gamma$ moments are available both in the kinetic [19] and the $1S$ scheme [3]. For the latter, HFAG uses the c-quark mass calculated in Ref. [20], $m_c^{\overline{\rm MS}}(3\ {\rm GeV}) = (0.998 \pm 0.029)$ GeV, obtained using low-energy sum rules. Note that the c-quark mass constraint cannot be applied in the $1S$ scheme as the $1S$ expressions do not depend on this parameter.

The results of the HFAG analysis in the kinetic scheme are given in Table 2 for both choices of the additional constraint. Note the excellent agreement in the b-quark mass, which is known from this study to almost 20 MeV precision. The relative uncertainty in $|V_{cb}|$ is about 1.7%. Table 3 contains the results in the $1S$ scheme for $B \to X_c\ell\nu$ only and using the $B \to X_s\gamma$ constraint. The central value of $|V_{cb}|$ is in excellent agreement with the kinetic scheme analysis. Due to a more aggressive error estimate, the relative precision here is 1.1%. The full result for all hadronic parameters and the entire correlation matrix is given in Ref. [21].

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